(1) The "Yukawa Potential" (10 Points)

The electric force dominates the interaction of particles because it is very long-range; i.e. the electrostatic potential falls off as an algebraic function of distance $1/r$. In certain circumstances we would like to "cut-off" the Coulomb field after some distance. This can be achieved using the “Yukawa” potential.

$$\Phi(r) = G \frac{ae^{-r/a}}{r},$$

where $G$ is some coupling constant, and $a$ is a distance. This potential can be mimicked in electrostatics by a point charge $q$ at the origin and "screened" by a smeared out negative charge distribution:

$$\rho(x) = -q \delta^{(3)}(x) + \frac{q}{4\pi a^2} e^{-r/a} \frac{1}{r}. \quad \text{(Here } a \text{ is a constant with dimensions of length)}$$

(a) Sketch $\Phi(r)$ showing the asymptotic form for $r<<a$ and $r>>a$. Explain the physical significance of the constant $a$.

(b) Show that the overall charge distribution is neutral. Does this make sense given the long range form of the potential?

(c) Derive the Yukawa potential for this charge distribution.

\textit{(Hint: Use Gauss' law to find the electric field first)}

(d) What is the potential energy stored in this charge distribution? Explain your result.
(2) **Potentials and contours** (15 points)
An infinitely long strip of width $L$ (shown below) carries a charge per unit area $\sigma$

(a) Find the electrostatic potential and the electric field in the $x$-$y$ plane.

(b) Take the following limits of $E$ and $\Phi$, and explain why the result is what you expect.

\[
\begin{align*}
\lim_{R \to 0} \frac{R}{L} \to 0 , & \quad (i) \quad \lim_{R \to \infty} \frac{R}{L} \to \infty , \\
\end{align*}
\]

where $R = \sqrt{x^2 + y^2}$

(c) Plot the field and equipotential contours. Do this 3 times with different ranges for the plots: (i) $x,y \sim L$. (ii) $x,y \gg L$. (iii) $x,y \ll L$. Explain your Results

(d) Now suppose there are two strips, one with positive surface charge $\sigma$ at $y=0$, and one with equal and opposite charge $-\sigma$, at $y = -s$ Use the principle of superposition to find $\Phi, E$

(e) Again plot the contours and electric field. Do this for two cases

\[
\begin{align*}
(i) \quad s \ll L . & \quad (ii) \quad s \gg L \quad \text{Explain your results.}
\end{align*}
\]
(3) **Boundary condition at a surface charge** (10 points)

(a) Each of the following geometries has a uniform charge density $\rho$. Find the electric field everywhere in space. Sketch the plot of $E$ as a function of the appropriate distance.

(i) Spherical shell, inner radius $a$, outer radius $b$,  
(ii) Cylindrical shell, length $L >> a, b$  
(iii) Infinite slab, thickness $d$

(b) Take the limit where thickness of each shell(slab) goes to zero, such that all of the charge is concentrated on a surface with density $\sigma$; i.e.,

(i) $b - a \rightarrow 0$, \hspace{1cm} $\rho(b - a) \rightarrow \sigma$  
(ii) $b - a \rightarrow 0$, \hspace{1cm} $\rho(b - a) \rightarrow \sigma$  
(iii) $d \rightarrow 0$, \hspace{1cm} $\rho d \rightarrow \sigma$

Show that the electric field normal to the surface is discontinuous by $\Delta E_\perp = 4\pi \sigma$

(4) **The Dirac Delta Function**: Jackson 1.3 (10 points)